On the Transient Response of an Infinite Cylindrical Antenna

The response of wide-angle conical and thin cylindrical antennas to dc pulse excitation, both in transmission and reception, has been reported.[1,2] One of the interesting remaining problems in this area is the determination of the time history of the radiation field of an infinite cylindrical antenna when a dc voltage pulse is applied across its input terminals.

One possible application of the results of such an investigation may be explained as follows: suppose that at the Nevada Test Site it is desired to measure the time history of the electromagnetic pulse emanating from a nuclear detonation. The instrumentation is believed to be properly calibrated, but this has not been verified. Clearly it would be desirable to have available a known transient electromagnetic field for checking oscilloscope triggering circuits and for making waveform observations to ascertain the extent of distortion introduced by the measuring equipment.[3] It appears that one way of developing an electromagnetic field in the form of a pulse of moderately long time duration is to select a wire with sufficient tensile strength to be supported by a jet helicopter. A pulse could be launched on the wire at its base, i.e., at the surface of the earth. From what has been learned from previous studies[1,2] it may be anticipated that a far-zone electromagnetic field that is essentially a replica of the wave shape of the applied driving voltage can be made to travel along the surface of the earth if a sufficiently long monopole is employed. For example, if the wire is 15 000 feet long, a pulse of rectangular or other shape a little less than 15.24-μs duration can be transmitted with essentially no waveform distortion.

An investigation of the pulse-radiating qualities of an infinite antenna is of considerable theoretical interest since the radiation field of a finite antenna is the same as that of an infinite antenna until the pulse reaches the free end of the finite structure. The far-zone field of an infinite cylindrical antenna of small radius is readily determined since it is represented by a simple formula in the frequency domain, and a high-speed digital computer is available for taking the Fourier transform. Evidently, a knowledge of the pulse handling capabilities of an infinite cylindrical antenna should illuminate the transient response of a finite but physically long cylindrical wave launcher, provided the applied voltage pulse is shorter than the time required for the pulse to travel (at the velocity of light) from the

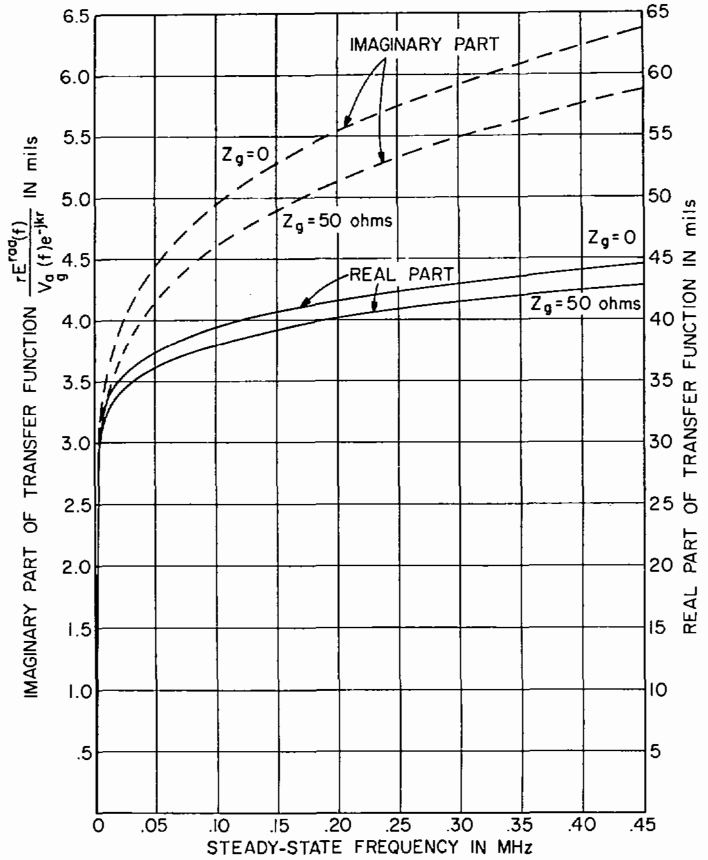

Fig. 1. The transfer functions given in (1) and (5).

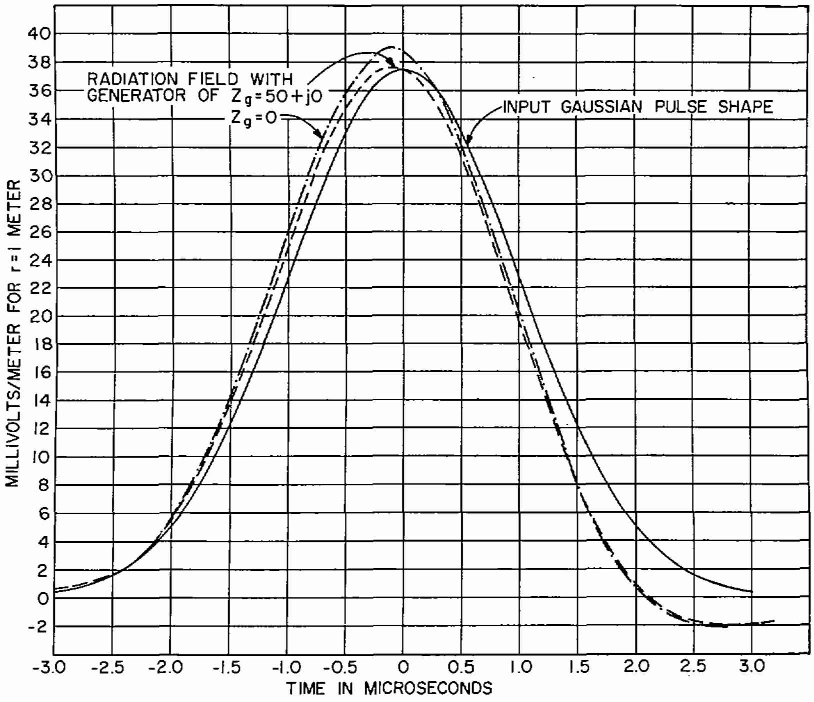

Fig. 2. The radiation field given in (9) and (10).

Manuscript received July 25, 1966; revised September 23, 1966.

[1] C. W. Harrison, Jr., and C. S. Williams, Jr., "Transients in wide-angle conical antennas," *IEEE Trans. on Antennas and Propagation*, vol. AP-13, pp. 236–246, March 1965.

[2] H. J. Schmitt, C. W. Harrison, Jr., and C. S. Williams, Jr., "Calculated and experimental response of thin cylindrical antennas to pulse excitation," *IEEE Trans. on Antennas and Propagation*, vol. AP-14, pp. 120–127, March 1966.

[3] C. W. Harrison, Jr., "Physical limitations on the measurement of transient fields in air and in dispersive media using electric and magnetic probes," *IEEE Trans. on Antennas and Propagation*, vol. AP-12, pp. 530–533, September 1964.

terminals to the free end of the structure.

A formula for the radiation field of an infinite perfectly conducting cylindrical antenna, i.e., the field that decreases with distance as $1/r$ where r is the radical distance measured normal to the axis of the antenna to the point of observation, has been derived by Papas.[4] It is

$$\frac{rE^{\text{rad}}(f)}{V_a(f)e^{-jkr}} = j\frac{1}{\pi}\left[\frac{1}{H_0{}^{(2)}(ka)}\right]. \tag{1}$$

Here $E^{\text{rad}}(f)$ is the far-zone electric field; $V_a(f)$ is the localized electromotive force acting on the cylinder of radius a in a narrow peripheral band at an arbitrary cross section; $k=2/\lambda$, where λ is the free-space wavelength. $H_0{}^{(2)}(x)$ is the Hankel function of zero order. The formula for $E^{\text{rad}}(f)$ is valid only when $ka \ll 1$. The assumed time dependence (but suppressed) is exp $(j\omega t)$. The magnetic field $H^{\text{rad}}(f)=E^{\text{rad}}(f)/\zeta_0$, where $\zeta_0=120\pi$ ohms. Since ζ_0 is a constant scaling factor, the time history of the magnetic field is identical to that of the electric field.

The driving point impedance of the antenna as given by King and Schmitt[5] is

$$Z_0(f) \doteq \frac{\zeta_0}{\pi}\Omega - j\frac{\zeta_0}{2} \tag{2}$$

where

$$\Omega = \ln\left(\frac{1}{ka}\right) - 0.5772. \tag{3}$$

This simple expression for $Z_0(f)$ is valid provided

$$\left(\frac{\pi}{\Omega}\right)^2 \ll 1. \tag{4}$$

If $V_g(f)$ is the generator voltage and $Z_g(f)$ is its internal impedance, (1) can be written as follows:

$$\frac{rE^{\text{rad}}(f)}{V_g(f)e^{-jkr}} = j\frac{Z_0(f)}{\pi[Z_g(f)+Z_0(f)]} \cdot \left[\frac{1}{H_0{}^{(2)}(ka)}\right]. \tag{5}$$

In the numerical work $v_a(t)$ and $v_g(t)$ are Gaussian pulses of the form

$$v(t) = Ae^{-(1/2)(t/t_1)^2} \tag{6}$$

where A is a constant scale factor. Hence,

$$\begin{aligned} V(f) &= A\int_{-\infty}^{\infty} e^{-(1/2)(t/t_1)^2} e^{-j2\pi ft}dt \\ &= At_1\sqrt{2\pi}\, e^{-(1/2)(f/f_1)^2} \end{aligned} \tag{7}$$

with

$$t_1 = \frac{1}{2\pi f_1}. \tag{8}$$

The truncation frequency employed in the Fourier integral was $f_c=2.6f_1$.[6] In all of the computations $t_1=1.0$ µs, $a=0.002$ m, and A in (6) was taken as 1 volt. When a generator with a nonzero internal impedance was used, the value selected was $Z_g=50+j0.0$. The transfer functions (1) and (5) were used in the transforms. Actually, the term exp (jkr) was omitted. If retardation is neglected the voltage $v_a(t)$ or $v_g(t)$ (appropriately adjusted in scale) can be superimposed on the electric field $e^{\text{rad}}(t)$. In the two cases considered, this permitted the direct observation of pulse distortion in transmission.

The expressions actually evaluated on the Sandia Laboratory CDC-3600 computer are the radiation field in the form

$$\begin{aligned} re^{\text{rad}}(t)10^3 &= 2 \times 10^3\sqrt{2\pi}\, t_1 \\ &\cdot \int_0^{2.6f_1} \exp\left(-\frac{f^2}{2f_1{}^2}\right)\frac{j}{\pi}\frac{1}{H_0{}^{(2)}(ka)} \\ &\cdot e^{j2\pi ft}df \end{aligned} \tag{9}$$

when the generator and antenna voltages are the same, and in the form

$$\begin{aligned} re^{\text{rad}}(t)10^3 &= 2 \times 10^3\sqrt{2\pi}\, t_1 \\ &\cdot \int_0^{2.6f_1} \frac{Z_0(f)}{Z_g(f)+Z_0(f)} \\ &\cdot \exp\left(-\frac{f^2}{2f_1{}^2}\right)\frac{j}{\pi}\frac{1}{H_0{}^{(2)}(ka)}e^{j2\pi ft}df \end{aligned} \tag{10}$$

when the generator has an internal impedance $Z_g(f)$.

The transfer function (1) or (5) was read out in each case. Graphical representations are shown in Fig. 1. It is seen that the transfer functions are reasonably flat with respect to frequency except at very low frequencies so that it may be anticipated that the wave shape of the radiation field should be a reasonable replica of the voltage pulse applied to the antenna. This is well confirmed by the curves in Fig. 2 where the computed fields are shown together with the true Gaussian pulse.

ACKNOWLEDGMENT

The problem was programmed for the Sandia Laboratory CDC-3600 computer by E. A. Aronson. The drawings were made by Dilla Tingley. The communication was proofread by Virginia Southerland. Barbra Ford typed the manuscript.

C. W. HARRISON, JR.
Sandia Corp.
Albuquerque, N. M.
R. W. P. KING
Gordon McKay Lab. of Appl. Sci.
Harvard University
Cambridge, Mass.

REFERENCE

[1] E. A. Lewis, J. E. Rasmussen, and J. R. Stahmann, "Waveforms and relative phase stability of transients radiated from a helicopter-supported antenna wire," *IEEE Trans. on Antennas and Propagation*, vol. AP-13, pp. 257–261, March 1965.

---

[4] C. H. Papas, "On the infinitely long cylindrical antenna," *J. Appl. Phys.*, vol. 20, pp. 437–440, May 1949.

[5] R. W. P. King and H. J. Schmitt, "The transient response of linear antennas and loops," *IRE Trans. on Antennas and Propagation*, vol. AP-10, pp. 222–228, May 1962.

[6] C. W. Harrison, Jr., "Transient electromagnetic field propagation through infinite sheets, into spherical shells, and into hollow cylinders," *IEEE Trans. on Antennas and Propagation*, vol. AP-12, pp. 319–334, May 1964.

The Input Admittance to a Slotted Array With or Without a Dielectric Sheet

The performance of a phased array is dependent upon the active driving point admittance of the array element as a function of scan. In fact, the scan limits of the array are set by the element mismatch rather than the grating lobe formation (in some cases, the large element mismatch is coincident with the appearance of the grating lobe). A number of papers have been written on the subject of the element match. One method [1], [2] gives a physical insight to the problem but gives qualitative rather than quantitative results. Another approach [3] which yields quantitative results, does not give physical insight to the problem and is very complicated. The interpretation given in Reference [4] of Stark's [5] derivation is useful because it gives one an understanding as to why the input admittance is expressed as a series. However, the interpretation is not valid when a dielectric sheet covers the array. In this paper, a transmission-line approach will be used to solve the problem. Because of the simplicity of this approach, it can be applied to more complicated problems, such as an array covered by several layers of dielectric sheets. In order to emphasize the approach rather than the solution to a particular problem, a two-dimensional problem will be solved.

The geometry of the problem is shown in Fig. 1. The array, which consists of a large number of slots, lies in the XY plane. Each slot is infinite in the Y direction and has a uniform electric field E, independent of Y in the X direction. The entire array is covered by a dielectric sheet of thickness t and relative permittivity ϵ_r. The array of slots is fed with a progressive phase shift v_0d so that the main beam is steered in the direction

$$\sin\theta_0 = \frac{\lambda v_0}{2\pi} = \frac{v_0}{k} \tag{1}$$

where $v_0=2\pi/\lambda \sin\theta_0$ and is the propagation constant in the X direction, k is the free-space propagation constant, and θ_0 is the direction of the peak of the main beam. The input admittance of one slot as a function of steering angle will be solved by the following method:

a) Express the field at $Z=0$ as the sum of an infinite number of plane waves.
b) Determine the input admittance at $Z=0$ for a plane wave.
c) Compute the total input power at $Z=0$ for an aperture having unit width in the Y direction and a length d in the X direction.
d) Express the input admittance in terms of the input power and voltage.

To express the aperture field as a sum of plane waves, we take the Fourier transform of the aperture distribution which is the "antenna pattern"

Manuscript received August 9, 1966; revised September 8, 1966. This work was supported under Contract AF30(602)-3886, Advanced Research Projects Agency Order 723.